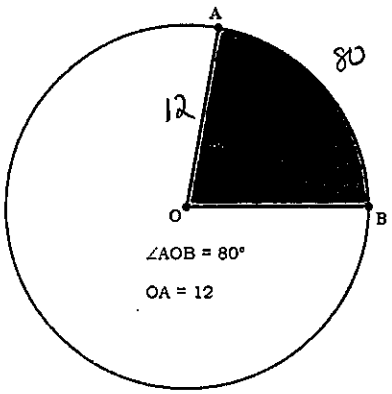
Notes – 11.6 – Arc Lengths and Areas of Sectors Name Key

Sector of a circle – a region bounded by two radii and an arc of the circle

Area of a sector:



$A = \pi \cdot 12^2$

$A = 144\pi$

$C = \pi \cdot 24$

$C = 24\pi$

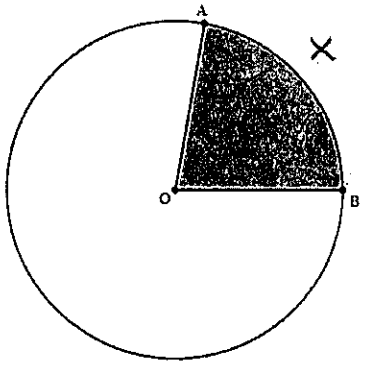
$\frac{80}{360}(24\pi) = 16.75$

$\frac{80}{360} \cdot 144\pi = 100.48$

If $m\widehat{AB} = x$, then:

length of $\widehat{AB} = \frac{x}{360} \cdot 2\pi r$ (portion of circle — of circle)

Area of sector AOB = $\frac{x}{360} \cdot \pi r^2$ (portion — Area)



Examples:

1. The radius of a circle is 3 cm. find (a) the lengths of the given arcs, and (b) the areas of the sectors determined by the given arcs. Use $\pi \approx \frac{22}{7}$.

a. 50°

a) $\frac{50}{360} \cdot 6 \cdot \frac{22}{7} = \frac{55}{21}$

b) $\frac{50}{360} \cdot 9 \cdot \frac{22}{7} = \frac{55}{14}$

b. 20°

a) $\frac{20}{360} \cdot 6 \cdot \frac{22}{7} = \frac{22}{21}$

b) $\frac{20}{360} \cdot 9 \cdot \frac{22}{7} = \frac{11}{7}$

c. 140°

a) $\frac{140}{360} \cdot 6 \cdot \frac{22}{7} = \frac{22}{3}$

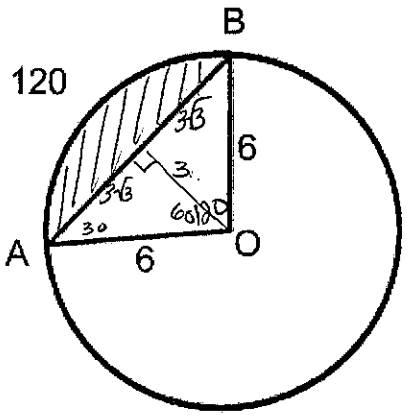
b) $\frac{140}{360} \cdot 9 \cdot \frac{22}{7} = \frac{11}{1}$

d. 210°

a) $\frac{210}{360} \cdot 6 \cdot \frac{22}{7} = \frac{11}{1}$

b) $\frac{210}{360} \cdot 9 \cdot \frac{22}{7} = \frac{33}{2}$

2. Find the area of the region bounded by $\overline{AB}$ and $\widehat{AB}$.



$A = \frac{120}{360} \cdot \pi \cdot 6^2 = \frac{36\pi}{3} = 12\pi$

$12\pi - \frac{1}{2}(3)(6\sqrt{3})$

$12\pi - 9\sqrt{3}$

3. Sector XPY is described by giving $m\angle XOY$ and the radius of circle O. make a sketch and find the length of $\widehat{XY}$ and the area of sector XOY.

	A	B	C	D	E
$m\angle XOY$	90	45	120	200	270
Radius	8	2	$6\sqrt{2}$	0.3	$\frac{8}{3}$

Length: $\frac{90}{360} \cdot \pi \cdot 16$ → 4π

$\frac{45}{360} \cdot \pi \cdot 4$ → $\frac{\pi}{2}$

$\frac{120}{360} \cdot \pi \cdot 6\sqrt{2}$ → $4\pi\sqrt{2}$

$\frac{200}{360} \cdot \pi \cdot 0.6$ → $\frac{3\pi}{9} = \frac{\pi}{3}$

$\frac{270}{360} \cdot \pi \cdot \frac{16}{3}$ → 4π

Area: $\frac{90}{360} \cdot \pi \cdot 64$ 16 → 16π

$\frac{45}{360} \cdot \pi \cdot 4$ 1 → $\frac{\pi}{2}$

$\frac{120}{360} \cdot \pi \cdot 72$ 24 → 24π

$\frac{200}{360} \cdot \pi \cdot .09$ 1 → $\frac{\pi}{20}$

105π

$\frac{270}{360} \cdot \pi \cdot \frac{64}{9}$ 16 3 → $\frac{16\pi}{3}$

4. The area of a sector APB is $\frac{5}{8}\pi$, and $m\angle AOB = 9$. Find the radius of Circle O.

$\frac{5}{45}\left(\frac{360}{9}\right)\frac{5}{8}\pi \quad \frac{9}{360} \cdot \pi r^2$

$25 = r^2$

$5 = r$