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5-1

Study Guide and Intervention

Monomials

Monomials A **monomial** is a number, a variable, or the product of a number and one or more variables. Constants are monomials that contain no variables.

Negative Exponent	$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ for any real number $a \neq 0$ and any integer n.

When you **simplify an expression**, you rewrite it without parentheses or negative exponents. The following properties are useful when simplifying expressions.

Product of Powers	$a^m \cdot a^n = a^{m+n}$ for any real number a and integers m and n.
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$ for any real number $a \neq 0$ and integers m and n.
Properties of Powers	For a, b real numbers and m, n integers: $(a^m)^n = a^{mn}$ $(ab)^m = a^m b^m$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ or $\frac{b^n}{a^n}$, $a \neq 0$, $b \neq 0$

Example **Simplify. Assume that no variable equals 0.**

a. $(3m^4n^{-2})(-5mn)^2$

$(3m^4n^{-2})(-5mn)^2 = 3m^4n^{-2} \cdot 25m^2n^2$
$= 75m^4m^2n^{-2}n^2$
$= 75m^{4+2}n^{-2+2}$
$= 75m^6$

b. $\frac{(-m^4)^3}{(2m^2)^{-2}}$

$\frac{(-m^4)^3}{(2m^2)^{-2}} = \frac{-m^{12}}{\frac{1}{4m^4}}$
$= -m^{12} \cdot 4m^4$
$= -4m^{16}$

Exercises

Simplify. Assume that no variable equals 0.

1. $c^{12} \cdot c^{-4} \cdot c^6$ c^{14}
2. $\frac{b^8}{b^2}$ b^6
3. $(a^4)^5$ a^{20}
4. $\frac{x^{-2}y}{x^4y^{-1}}$ $\frac{y^2}{x^6}$
5. $\left(\frac{a^2b}{a^{-3}b^2}\right)^{-1}$ $\frac{b}{a^5}$
6. $\left(\frac{x^2y}{xy^3}\right)^2$ $\frac{x^2}{y^4}$
7. $\frac{1}{5}(-5a^2b^3)^2(abc)^2$ $5a^6b^8c^2$
8. $m^7 \cdot m^8$ m^{15}
9. $\frac{8m^3n^2}{4mn^3}$ $\frac{2m^2}{n}$
10. $\frac{2^3c^4t^2}{2^2c^4t^2}$ 2
11. $4j(2j^{-2}k^2)(3j^3k^{-7})$ $\frac{24j^2}{k^5}$
12. $\frac{2mn^2(3m^2n)^2}{12m^3n^4}$ $\frac{3}{2}m^2$

Lesson 5-1

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5-1

Study Guide and Intervention *(continued)*

Monomials

Scientific Notation

Scientific notation	A number expressed in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer

Example 1 **Express 46,000,000 in scientific notation.**

$46{,}000{,}000 = 4.6 \times 10{,}000{,}000$ $1 \leq 4.6 < 10$
$= 4.6 \times 10^7$ Write 10,000,000 as a power of ten.

Example 2 **Evaluate $\frac{3.5 \times 10^4}{5 \times 10^{-2}}$. Express the result in scientific notation.**

$\frac{3.5 \times 10^4}{5 \times 10^{-2}} = \frac{3.5}{5} \times \frac{10^4}{10^{-2}}$
$= 0.7 \times 10^6$
$= 7 \times 10^5$

Exercises

Express each number in scientific notation.

1. 24,300 — 2.43×10^4
2. 0.00099 — 9.9×10^{-4}
3. 4,860,000 — 4.86×10^6
4. 525,000,000 — 5.25×10^8
5. 0.0000038 — 3.8×10^{-6}
6. 221,000 — 2.21×10^5
7. 0.000000064 — 6.4×10^{-8}
8. 16,750 — 1.675×10^4
9. 0.000369 — 3.69×10^{-4}

Evaluate. Express the result in scientific notation.

10. $(3.6 \times 10^4)(5 \times 10^3)$ — 1.8×10^8
11. $(1.4 \times 10^{-8})(8 \times 10^{12})$ — 1.12×10^5
12. $(4.2 \times 10^{-3})(3 \times 10^{-2})$ — 1.26×10^{-4}
13. $\frac{9.5 \times 10^7}{3.8 \times 10^{-2}}$ — 2.5×10^9
14. $\frac{1.62 \times 10^{-2}}{1.8 \times 10^5}$ — 9×10^{-8}
15. $\frac{4.81 \times 10^8}{6.5 \times 10^4}$ — 7.4×10^3
16. $(3.2 \times 10^{-3})^2$ — 1.024×10^{-5}
17. $(4.5 \times 10^7)^2$ — 2.025×10^{15}
18. $(6.8 \times 10^{-5})^2$ — 4.624×10^{-9}

19. **ASTRONOMY** Pluto is 3,674.5 million miles from the sun. Write this number in scientific notation. Source: *New York Times Almanac* 3.6745×10^9 miles

20. **CHEMISTRY** The boiling point of the metal tungsten is 10,220°F. Write this temperature in scientific notation. Source: *New York Times Almanac* 1.022×10^4

21. **BIOLOGY** The human body contains 0.0004% iodine by weight. How many pounds of iodine are there in a 120-pound teenager? Express your answer in scientific notation. Source: *Universal Almanac* 4.8×10^{-4} lb

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5-2

Study Guide and Intervention

Polynomials

Add and Subtract Polynomials

Polynomial	a monomial or a sum of monomials
Like Terms	terms that have the same variable(s) raised to the same power(s)

To add or subtract polynomials, perform the indicated operations and combine like terms.

Example 1 **Simplify $-6rs + 18r^2 - 5s^2 - 14r^2 + 8rs - 6s^2$.**

$-6rs + 18r^2 - 5s^2 - 14r^2 + 8rs - 6s^2$
$= (18r^2 - 14r^2) + (-6rs + 8rs) + (-5s^2 - 6s^2)$ Group like terms.
$= 4r^2 + 2rs - 11s^2$ Combine like terms.

Example 2 **Simplify $4xy^2 + 12xy - 7x^2y - (20xy + 5xy^2 - 8x^2y)$.**

$4xy^2 + 12xy - 7x^2y - (20xy + 5xy^2 - 8x^2y)$
$= 4xy^2 + 12xy - 7x^2y - 20xy - 5xy^2 + 8x^2y$ Distribute the minus sign.
$= (-7x^2y + 8x^2y) + (4xy^2 - 5xy^2) + (12xy - 20xy)$ Group like terms.
$= x^2y - xy^2 - 8xy$ Combine like terms.

Exercises

Simplify.

1. $(6x^2 - 3x + 2) - (4x^2 + x - 3)$
   $2x^2 - 4x + 5$
2. $(7y^2 + 12xy - 5x^2) + (6xy - 4y^2 - 3x^2)$
   $3y^2 + 18xy - 8x^2$
3. $(-4m^2 - 6m) - (6m + 4m^2)$
   $-8m^2 - 12m$
4. $27x^2 - 5y^2 + 12y^2 - 14x^2$
   $13x^2 + 7y^2$
5. $(18p^2 + 11pq - 6q^2) - (15p^2 - 3pq + 4q^2)$
   $3p^2 + 14pq - 10q^2$
6. $17j^2 - 12k^2 + 3j^2 - 15j^2 + 14k^2$
   $5j^2 + 2k^2$
7. $(8m^2 - 7n^2) - (n^2 - 12m^2)$
   $20m^2 - 8n^2$
8. $14bc + 6b - 4c + 8b - 8c + 8bc$
   $14b + 22bc - 12c$
9. $6r^2s + 11rs^2 + 3r^2s - 7rs^2 + 15r^2s - 9rs^2$
   $24r^2s - 5rs^2$
10. $-9xy + 11x^2 - 14y^2 - (6y^2 - 5xy - 3x^2)$
    $14x^2 - 4xy - 20y^2$
11. $(12xy - 8x + 3y) + (15x - 7y - 8xy)$
    $7x + 4xy - 4y$
12. $10.8b^2 - 5.7b + 7.2 - (2.9b^2 - 4.6b - 3.1)$
    $7.9b^2 - 1.1b + 10.3$
13. $(3bc - 9b^2 - 6c^2) + (4c^2 - b^2 + 5bc)$
    $-10b^2 + 8bc - 2c^2$
14. $11x^2 + 4y^2 + 6xy + 3y^2 - 5xy - 10x^2$
    $x^2 + xy + 7y^2$
15. $\frac{1}{4}x^2 - \frac{3}{8}xy + \frac{1}{2}y^2 - \frac{1}{2}xy + \frac{1}{4}y^2 - \frac{3}{8}x^2$
    $-\frac{1}{8}x^2 - \frac{7}{8}xy + \frac{3}{4}y^2$
16. $24p^3 - 15p^2 + 3p - 15p^3 + 13p^2 - 7p$
    $9p^3 - 2p^2 - 4p$

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5-2

Study Guide and Intervention *(continued)*

Polynomials

Multiply Polynomials You use the distributive property when you multiply polynomials. When multiplying binomials, the **FOIL** pattern is helpful.

FOIL Pattern	To multiply two binomials, add the products of F the *first* terms, O the *outer* terms, I the *inner* terms, and L the *last* terms.

Example 1 **Find $4y(6 - 2y + 5y^2)$.**

$4y(6 - 2y + 5y^2) = 4y(6) + 4y(-2y) + 4y(5y^2)$ Distributive Property
$= 24y - 8y^2 + 20y^3$ Multiply the monomials.

Example 2 **Find $(6x - 5)(2x + 1)$.**

$(6x - 5)(2x + 1) = 6x \cdot 2x + 6x \cdot 1 + (-5) \cdot 2x + (-5) \cdot 1$
(First terms, Outer terms, Inner terms, Last terms)
$= 12x^2 + 6x - 10x - 5$ Multiply monomials.
$= 12x^2 - 4x - 5$ Add like terms.

Exercises

Find each product.

1. $2x(3x^2 - 5)$
   $6x^3 - 10x$
2. $7a(6 - 2a - a^2)$
   $42a - 14a^2 - 7a^3$
3. $-5y^2(y^2 + 2y - 3)$
   $-5y^4 - 10y^3 + 15y^2$
4. $(x - 2)(x + 7)$
   $x^2 + 5x - 14$
5. $(5 - 4x)(3 - 2x)$
   $15 - 22x + 8x^2$
6. $(2x - 1)(3x + 5)$
   $6x^2 + 7x - 5$
7. $(4x + 3)(x + 8)$
   $4x^2 + 35x + 24$
8. $(7x - 2)(2x - 7)$
   $14x^2 - 53x + 14$
9. $(3x - 2)(x + 10)$
   $3x^2 + 28x - 20$
10. $3(2a + 5c) - 2(4a - 6c)$
    $-2a + 27c$
11. $2(a - 6)(2a + 7)$
    $4a^2 - 10a - 84$
12. $2x(x + 5) - x^2(3 - x)$
    $x^3 - x^2 + 10x$
13. $(3t^2 - 8)(t^2 + 5)$
    $3t^4 + 7t^2 - 40$
14. $(2r + 7)^2$
    $4r^2 + 28r + 49$
15. $(c + 7)(c - 3)$
    $c^2 + 4c - 21$
16. $(5a + 7)(5a - 7)$
    $25a^2 - 49$
17. $(3x^2 - 1)(2x^2 + 5x)$
    $6x^4 + 15x^3 - 2x^2 - 5x$
18. $(x^2 - 2)(x^3 - 5)$
    $x^4 - 7x^2 + 10$
19. $(x + 1)(2x^2 - 3x + 1)$
    $2x^3 - x^2 - 2x + 1$
20. $(2n^2 - 3)(n^2 + 5n - 1)$
    $2n^4 + 10n^3 - 5n^2 - 15n + 3$
21. $(x - 1)(x^2 - 3x + 4)$
    $x^3 - 4x^2 + 7x - 4$

Answers

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5-3

Study Guide and Intervention

Dividing Polynomials

Use Long Division To divide a polynomial by a monomial, use the properties of powers from Lesson 5-1.

To divide a polynomial by a polynomial, use a long division pattern. Remember that only like terms can be added or subtracted.

Example 1 Simplify $\frac{12p^3t^2r - 21p^2qtr^2 - 9p^3tr}{3p^2tr}$.

$$\frac{12p^3t^2r - 21p^2qtr^2 - 9p^3tr}{3p^2tr} = \frac{12p^3t^2r}{3p^2tr} - \frac{21p^2qtr^2}{3p^2tr} - \frac{9p^3tr}{3p^2tr}$$

$$= \frac{12}{3}p^{3-2}t^{2-1}r^{1-1} - \frac{21}{3}p^{2-2}qt^{1-1}r^{2-1} - \frac{9}{3}p^{3-2}t^{1-1}r^{1-1}$$

$$= 4pt - 7qr - 3p$$

Example 2 Use long division to find $(x^3 - 8x^2 + 4x - 9) \div (x - 4)$.

$$\begin{array}{r} x^2 - 4x - 12 \\ x - 4 \overline{)\, x^3 - 8x^2 + 4x - 9} \\ \underline{(-)x^3 - 4x^2} \\ -4x^2 + 4x \\ \underline{(-)-4x^2 + 16x} \\ -12x - 9 \\ \underline{(-)-12x + 48} \\ -57 \end{array}$$

The quotient is $x^2 - 4x - 12$, and the remainder is -57.

Therefore $\frac{x^3 - 8x^2 + 4x - 9}{x - 4} = x^2 - 4x - 12 - \frac{57}{x - 4}$.

Exercises

Simplify.

1. $\frac{18a^3 + 30a^2}{3a}$ $6a^2 + 10a$

2. $\frac{24mn^6 - 40m^2n^3}{4m^2n^3}$ $\frac{6n^3}{m} - 10$

3. $\frac{60a^2b^3 - 48b^4 + 84a^5b^2}{12ab^2}$ $5ab - \frac{4b^2}{a} + 7a^4$

4. $(2x^2 - 5x - 3) \div (x - 3)$ $2x + 1$

5. $(m^2 - 3m - 7) \div (m + 2)$ $m - 5 + \frac{3}{m + 2}$

6. $(p^3 - 6) \div (p - 1)$ $p^2 + p + 1 - \frac{5}{p - 1}$

7. $(t^3 - 6t^2 + 1) \div (t + 2)$ $t^2 - 8t + 16 - \frac{31}{t + 2}$

8. $(x^5 - 1) \div (x - 1)$ $x^4 + x^3 + x^2 + x + 1$

9. $(2x^3 - 5x^2 + 4x - 4) \div (x - 2)$ $2x^2 - x + 2$

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5-3

Study Guide and Intervention (continued)

Dividing Polynomials

Use Synthetic Division

Synthetic division	a procedure to divide a polynomial by a binomial using coefficients of the dividend and the value of r in the divisor $x - r$

Use synthetic division to find $(2x^3 - 5x^2 + 5x - 2) \div (x - 1)$.

Step 1	Write the terms of the dividend so that the degrees of the terms are in descending order. Then write just the coefficients.	$2x^3 - 5x^2 + 5x - 2$ 2 −5 5 −2
Step 2	Write the constant r of the divisor $x - r$ to the left. In this case, $r = 1$. Bring down the first coefficient, 2, as shown.	1 | 2 −5 5 −2 2
Step 3	Multiply the first coefficient by r, $1 \cdot 2 = 2$. Write their product under the second coefficient. Then add the product and the second coefficient: $-5 + 2 = -3$.	1 | 2 −5 5 −2 2 2 −3
Step 4	Multiply the sum, -3, by r: $-3 \cdot 1 = -3$. Write the product under the next coefficient and add: $5 + (-3) = 2$.	1 | 2 −5 5 −2 2 −3 2 −3 2
Step 5	Multiply the sum, 2, by r: $2 \cdot 1 = 2$. Write the product under the next coefficient and add: $-2 + 2 = 0$. The remainder is 0.	1 | 2 −5 5 −2 2 −3 2 2 −3 2 0

Thus, $(2x^3 - 5x^2 + 5x - 2) \div (x - 1) = 2x^2 - 3x + 2$.

Exercises

Simplify.

1. $(3x^3 - 7x^2 + 9x - 14) \div (x - 2)$ $3x^2 - x + 7$

2. $(5x^3 + 7x^2 - x - 3) \div (x + 1)$ $5x^2 + 2x - 3$

3. $(2x^3 + 3x^2 - 10x - 3) \div (x + 3)$ $2x^2 - 3x - 1$

4. $(x^3 - 8x^2 + 19x - 9) \div (x - 4)$ $x^2 - 4x + 3 + \frac{3}{x - 4}$

5. $(2x^3 + 10x^2 + 9x + 38) \div (x + 5)$ $2x^2 + 9 - \frac{7}{x + 5}$

6. $(3x^3 - 8x^2 + 16x - 1) \div (x - 1)$ $3x^2 - 5x + 11 + \frac{10}{x - 1}$

7. $(x^3 - 9x^2 + 17x - 1) \div (x - 2)$ $x^2 - 7x + 3 + \frac{5}{x - 2}$

8. $(4x^3 - 25x^2 + 4x + 20) \div (x - 6)$ $4x^2 - x - 2 + \frac{8}{x - 6}$

9. $(6x^3 + 28x^2 - 7x + 9) \div (x + 5)$ $6x^2 - 2x + 3 - \frac{6}{x + 5}$

10. $(x^4 - 4x^3 + x^2 + 7x - 2) \div (x - 2)$ $x^3 - 2x^2 - 3x + 1$

11. $(12x^4 + 20x^3 - 24x^2 + 20x + 35) \div (3x + 5)$ $4x^3 - 8x + 20 + \frac{-65}{3x + 5}$

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5-4 Study Guide and Intervention

Factoring Polynomials

Factor Polynomials

Techniques for Factoring Polynomials	For any number of terms, check for: greatest common factor
	For two terms, check for: Difference of two squares $a^2 - b^2 = (a + b)(a - b)$ Sum of two cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ Difference of two cubes $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
	For three terms, check for: Perfect square trinomials $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$ General trinomials $acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$
	For four terms, check for: Grouping $ax + bx + ay + by = x(a + b) + y(a + b)$ $= (a + b)(x + y)$

Example **Factor $24x^2 - 42x - 45$.**

First factor out the GCF to get $24x^2 - 42x - 45 = 3(8x^2 - 14x - 15)$. To find the coefficients of the x terms, you must find two numbers whose product is $8 \cdot (-15) = -120$ and whose sum is -14. The two coefficients must be -20 and 6. Rewrite the expression using $-20x$ and $6x$ and factor by grouping.

$8x^2 - 14x - 15 = 8x^2 - 20x + 6x - 15$ Group to find a GCF.
$= 4x(2x - 5) + 3(2x - 5)$ Factor the GCF of each binomial.
$= (4x + 3)(2x - 5)$ Distributive Property

Thus, $24x^2 - 42x - 45 = 3(4x + 3)(2x - 5)$.

Exercises

Factor completely. If the polynomial is not factorable, write *prime*.

1. $14x^2y^2 + 42xy^3$
   $14xy^2(x + 3y)$
2. $6mn + 18m - n - 3$
   $(6m - 1)(n + 3)$
3. $2x^2 + 18x + 16$
   $2(x + 8)(x + 1)$
4. $x^4 - 1$
   $(x^2 + 1)(x + 1)(x - 1)$
5. $35x^3y^4 - 60x^4y$
   $5x^3y(7y^3 - 12x)$
6. $2r^3 + 250$
   $2(r + 5)(r^2 - 5r + 25)$
7. $100m^8 - 9$
   $(10m^4 - 3)(10m^4 + 3)$
8. $x^2 + x + 1$
   prime
9. $c^4 + c^3 - c^2 - c$
   $c(c + 1)^2(c - 1)$

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5-4 Study Guide and Intervention *(continued)*

Factoring Polynomials

Simplify Quotients In the last lesson you learned how to simplify the quotient of two polynomials by using long division or synthetic division. Some quotients can be simplified by using factoring.

Example **Simplify $\frac{8x^2 + 11x + 12}{2x^2 - 13x - 24}$.**

$\frac{8x^2 + 11x + 12}{2x^2 - 13x - 24} = \frac{(2x + 3)(x + 4)}{(x - 8)(2x + 3)}$ Factor the numerator and denominator.

$= \frac{x + 4}{x - 8}$ Divide. Assume $x \neq 8, -\frac{3}{2}$.

Exercises

Simplify. Assume that no denominator is equal to 0.

1. $\frac{x^2 - 7x + 12}{x^2 - x - 6}$ $\frac{x - 4}{x + 2}$
2. $\frac{x^2 + 6x + 5}{2x^2 - x - 3}$ $\frac{x + 5}{2x - 3}$
3. $\frac{x^2 - 11x + 30}{x^2 - 5x - 6}$ $\frac{x - 5}{x + 1}$
4. $\frac{x^2 + x - 6}{x^2 - 7x + 10}$ $\frac{x + 3}{x - 5}$
5. $\frac{2x^2 + 5x - 3}{4x^2 + 11x - 3}$ $\frac{2x - 1}{4x - 1}$
6. $\frac{5x^2 + 9x - 2}{x^2 + 5x + 6}$ $\frac{5x - 1}{x + 3}$
7. $\frac{4x^2 + 4x - 3}{2x^2 - x - 6}$ $\frac{2x - 1}{x - 2}$
8. $\frac{6x^2 + 25x + 4}{x^2 + 6x + 8}$ $\frac{6x + 1}{x + 2}$
9. $\frac{x^2 - 7x + 10}{3x^2 - 8x - 35}$ $\frac{x - 2}{3x + 7}$
10. $\frac{4x^2 + 16x + 15}{2x^2 + x - 3}$ $\frac{2x + 5}{x - 1}$
11. $\frac{3x^2 + 4x - 15}{2x^2 + 3x - 9}$ $\frac{3x - 5}{2x - 3}$
12. $\frac{x^2 - 14x + 49}{x^2 - 2x - 35}$ $\frac{x - 7}{x + 5}$
13. $\frac{x^2 - 81}{2x^2 - 23x + 45}$ $\frac{x + 9}{2x - 5}$
14. $\frac{7x^2 + 11x - 6}{x^2 - 4}$ $\frac{7x - 3}{x - 2}$
15. $\frac{4x^2 - 12x + 9}{2x^2 + 13x - 24}$ $\frac{2x - 3}{x + 8}$
16. $\frac{4x^2 - 4x - 3}{8x^3 + 1}$ $\frac{2x - 3}{4x^2 - 2x + 1}$
17. $\frac{y^3 - 64}{3y^2 - 17y + 20}$ $\frac{y^2 + 4y + 16}{3y - 5}$
18. $\frac{27x^3 - 8}{9x^2 - 4}$ $\frac{9x^2 + 6x + 4}{3x + 2}$

Answers