12-3

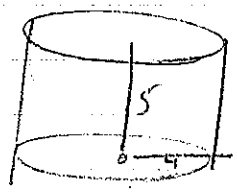
Cylinders:

$LA = 2\pi r \cdot h$ $\quad V = \pi r^2 \cdot h$

Cones:

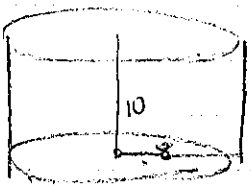
$LA = \frac{1}{2}(2\pi r)l = \pi r l$ $\quad V = \frac{1}{3}(\pi r^2)h$

(1)



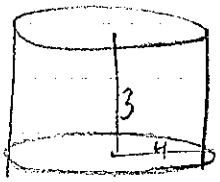
$LA = 8\pi \cdot 5 = 40\pi$
$TA = 40\pi + 2(\pi)(4^2) = 72\pi$
$V = \pi(4^2) \cdot 5 = 80\pi$

(2)



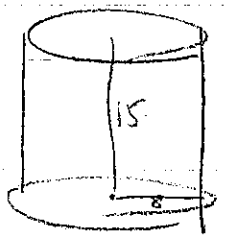
$LA = 16\pi(10) = 160\pi$
$TA = 160\pi + 2(8^2)\pi = 288\pi$
$V = \pi \cdot 8^2 \cdot 10 = 640\pi$

(3)



$LA = 8\pi \cdot 3 = 24\pi$
$TA = 24\pi + 2 \cdot 16\pi = 56\pi$
$V = \pi \cdot 4^2 \cdot 3 = 48\pi$

(4)



$LA = 16\pi \cdot 15 = 240\pi$
$TA = 240\pi + 264\pi = 368\pi$
$V = 64\pi \cdot 15 = 960\pi$

(5) $V = \pi r^2 h$

$64\pi = \pi r^2 \cdot r$

$64 = r^3$

$4 = r$

(6) $LA = 2\pi r \cdot h$

$18\pi = 2\pi r \cdot 6$

$18 = 12r$

$1.5 = r$

(7) $V = \pi r^2 h$

$72\pi = \pi r^2 \cdot 8$

$9 = r^2$

$3 = r$

$LA = 2\pi r \cdot h$

$LA = 2\pi \cdot 3 \cdot 8$

$LA = 48\pi$

(8) $TA = LA + 2B$

$100\pi = 2\pi \cdot h \cdot h + 2(\pi h^2)$

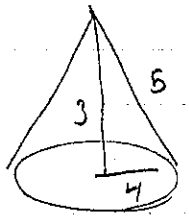
$100\pi = 2h^2\pi + 2h^2\pi$

$100 = 4h^2$

$5 = h = r$

	r	h	ℓ	LA	TA	V
(9)	4	3	5	20π	36π	16π
(10)	8	6	10	80π	144π	128π
(11)	12	5	13	156π	300π	240π
(12)		2	6			
(13)			15	180π		
(14)	21			609π		
(15)	15					600π
(16)	9					324π

(9)



$4^2 + 3^2 = \ell^2$

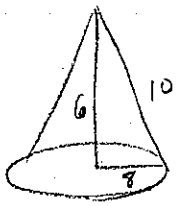
$5 = \ell$

$LA = \frac{1}{2} \cdot \pi \cdot 8 \cdot 5 = 20\pi$

$TA = 20\pi + \pi \cdot 16 = 36\pi$

$V = \frac{1}{3}\pi \cdot 4^2 \cdot 3 = 16\pi$

(10)



$8^2 + 6^2 = \ell^2$

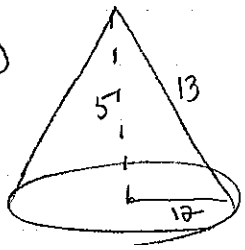
$10 = \ell$

$LA = \frac{1}{2}\pi \cdot 16 \cdot 10 = 80\pi$

$TA = 80\pi + 64\pi = 144\pi$

$V = \frac{1}{3}\pi \cdot 64 \cdot 6 = 128\pi$

(11)



$h^2 + 12^2 = 13^2$

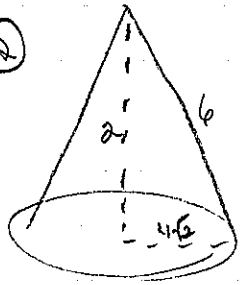
$h = 5$

$LA = \frac{1}{2}\pi \cdot 24 \cdot 13 = 156\pi$

$TA = 156\pi + 144\pi = 300\pi$

$V = \frac{1}{3} 144\pi \cdot 5 = 240\pi$

(12)



$2^2 + r^2 = 6^2$

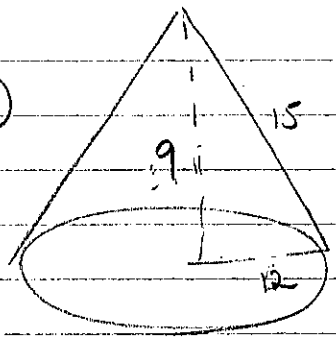
$r = \sqrt{32} = 4\sqrt{2}$

$LA = \frac{1}{2} \cdot 8\sqrt{2}\pi \cdot 6 = 24\pi\sqrt{2}$

$TA = 24\pi\sqrt{2} + (4\sqrt{2})^2\pi = 24\pi\sqrt{2} + 32\pi$

$V = \frac{1}{3}\pi(4\sqrt{2})^2 \cdot 2 = \frac{64\pi}{3}$

(13)



$LA = 180\pi$

$TA = 180\pi + 144\pi = \boxed{324\pi}$

$V = \frac{1}{3} \cdot 144\pi \cdot 9$

$= \boxed{432\pi}$

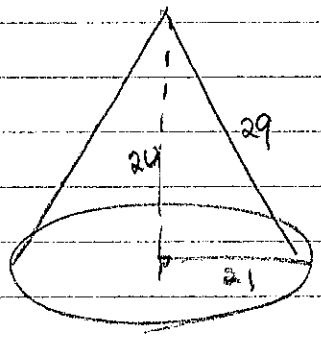
$180\pi = \frac{1}{2} 2\pi r \cdot 15$

$\boxed{12 = r}$

$12^2 + h^2 = 15^2$

$\boxed{h = 9}$

(14)



$LA = 609\pi$

$TA = 609\pi + 441\pi = \boxed{1050\pi}$

$V = \frac{1}{3} \cdot 441\pi \cdot 20$

$= 2940\pi$

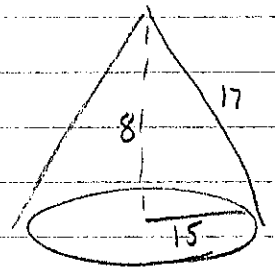
$609\pi = \frac{1}{2} \cdot 42\pi \cdot l$

$\boxed{29 = l}$

$21^2 + h^2 = 29^2$

$\boxed{h = 20}$

(15)



$V = 600\pi$

$LA = \frac{1}{2} \cdot 30\pi \cdot 17 = 255\pi$

$TA = 255\pi + 225\pi = 480\pi$

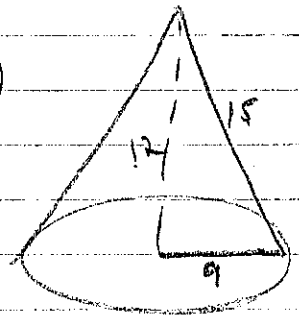
$600\pi = \frac{1}{3} \cdot \pi \cdot 15^2 \cdot h$

$\boxed{8 = h}$

$8^2 + 15^2 = l^2$

$\boxed{17 = l}$

(16)



$V = 324\pi$

$LA = \frac{1}{2} \cdot 18\pi \cdot 15 = 135\pi$

$TA = 135\pi + 81\pi = 216\pi$

$324\pi = \frac{1}{3} \cdot \pi \cdot 9^2 \cdot h$

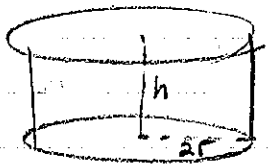
$\boxed{12 = h}$

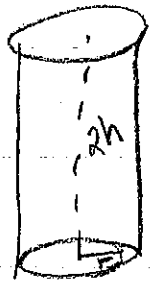
$12^2 + 9^2 = l^2$

$\boxed{15 = l}$

(17) a) $\frac{20\pi}{80\pi} = 1:4$ b) $\frac{36\pi}{144\pi} = 1:4$ c) $\frac{16\pi}{128\pi} = 1:8$

(18)

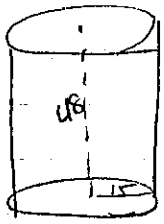


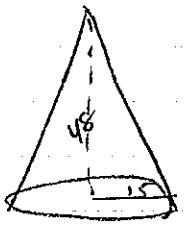


$\frac{\text{short}}{\text{tall}} = \frac{\pi(2r)^2 \cdot h}{\pi(r^2) \cdot 2h} = \frac{4\pi r^2 h}{2\pi r^2 h} = \frac{2}{1}$

Wider holds more → 2:1

(19)

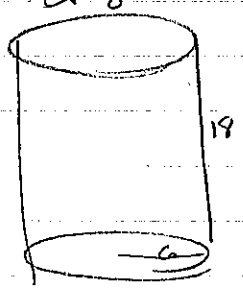




3:1

(20)

(21)

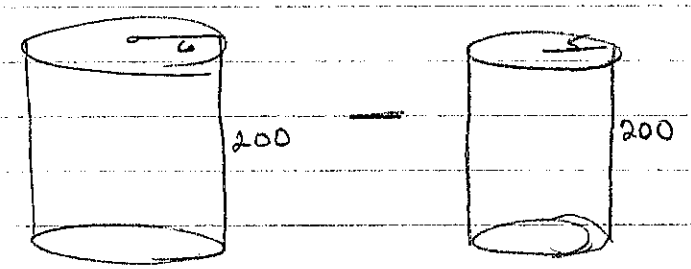


$V = 36\pi \cdot 18 = \boxed{648\pi}$

$648\pi = \frac{1}{3} \cdot \pi \cdot 9^2 \cdot h$

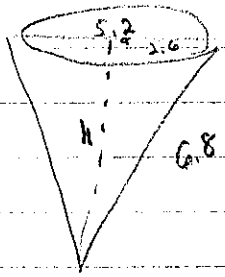
$\boxed{24 = h}$

(22)



$7200\pi - 5000\pi = 2200\pi \text{ cm}^3 = \boxed{6908 \text{ cm}^3}$

(23)



$V = \frac{1}{3} \cdot 2.6^2\pi \cdot 6.28$

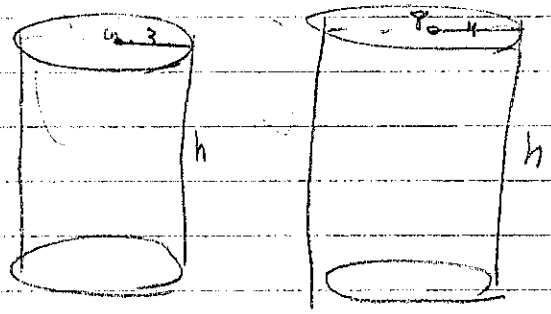
$V = 44.4 \text{ cm}^3$

$2.6^2 + h^2 = 6.8^2$

$h = 6.28$

$\frac{44.4}{1.8} = \boxed{24.6 \text{ min.}}$

(24)



$9\pi \cdot h + 16\pi \cdot h = r^2\pi \cdot h$

$25 = r^2$

$5 = r \longrightarrow \boxed{10 = d}$

(25) $40\pi = 2 \cdot \pi r^2 + 2\pi r \cdot 8$

$40\pi = 2\pi r^2 + 16\pi r$

$40 = 2r^2 + 16r$

$0 = 2r^2 + 16r - 40$

$0 = 2(r^2 + 8r - 20)$

$0 = 2(r+10)(r-2)$

$\boxed{r = 2}$

(26) $TA = 90\pi$

$h = 12$

$r = ?$

$90\pi = 2\pi r^2 + 2\pi r(12)$

$90\pi = 2\pi r^2 + 24\pi r$

$90 = 2r^2 + 24r$

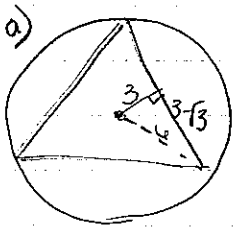
$2r^2 + 24r - 90 = 0$

$2(r^2 + 12r - 45) = 0$

$2(r+15)(r-3) = 0$

$\boxed{r = 3}$

(29) a)



$s = 6\sqrt{3}$

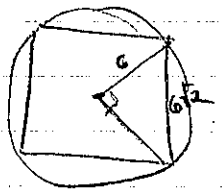
$a = 3$

$A_\Delta = \frac{1}{2}(18\sqrt{3})(3) = 27\sqrt{3}$

$LA = 18\sqrt{3} \cdot 10 = \boxed{180\sqrt{3}}$

$V = \frac{1}{2}(18\sqrt{3})(3)(10) = \boxed{270\sqrt{3}}$

b)



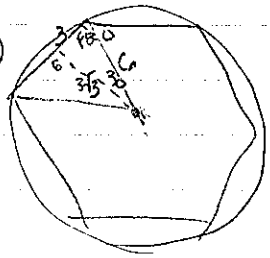
$s = 6\sqrt{2}$

$A_\square = 72$

$LA = 24\sqrt{2} \cdot 10 = \boxed{240\sqrt{2}}$

$V = 72 \cdot 10 = \boxed{720}$

c)



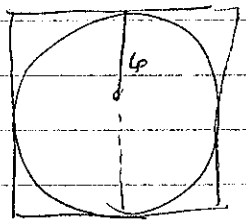
$a = 3\sqrt{3}$

$p = 36$

$A_{hex} = \frac{1}{2} \cdot 3\sqrt{3} \cdot 36 = 54\sqrt{3}$

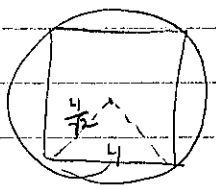
$LA = 36 \cdot 10 = 360$

$V = 54\sqrt{3} \cdot 10 = 540\sqrt{3}$

(32) 

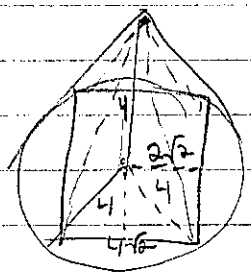
$h = 10$ $\quad V = 144 \cdot 10 = \boxed{1440}$

$S = 12$

(33) 

$h = 6$ $\quad V = \frac{1}{3}\pi (2\sqrt{2})^2 \cdot 6 = \frac{1}{3}\pi \cdot 8 = \boxed{16\pi}$

$r = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

(34) 

$h = 4$ $\quad$ a) $V_{cone} = \frac{1}{3}(4\sqrt{2})^2 \cdot 4 = \frac{128}{3}$

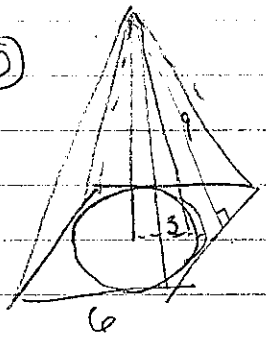
$S = 4\sqrt{2}$ $\quad$ b) Cone: $4^2 + 4^2 = l^2$

$\sqrt{32} = \boxed{l = 4\sqrt{2}}$

pyramid: $4^2 + (2\sqrt{2})^2 = l^2$

$16 + 8 = l^2$

$\sqrt{24} = \boxed{l = 2\sqrt{6}}$

(35) 

$h^2 + 3^2 = 9^2$ $\quad V_{cone} = \frac{1}{3}\pi (3)^2 \cdot 6\sqrt{2}$

$h = \sqrt{72} = 6\sqrt{2}$ $\quad V_{cone} = 18\pi\sqrt{2}$

$r = 3$

(36) $LA = \frac{3}{5} TA$,

$\frac{1}{2} \cdot 2\pi r \cdot l = \frac{3}{5}\left(\frac{1}{2} 2\pi r l + \pi r^2\right)$

$5\left(\pi r l = \frac{3}{5}\pi r l + \frac{3}{5}\pi r^2\right)$

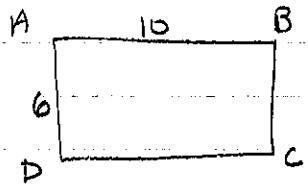
$5\pi r l = 3\pi r l + 3\pi r^2$

$2\pi r l = 3\pi r^2$

$\frac{2l}{3l} = \frac{3r}{3l}$ $\longrightarrow$ $\boxed{\frac{2}{3} = \frac{r}{l}}$

12.3

(27)

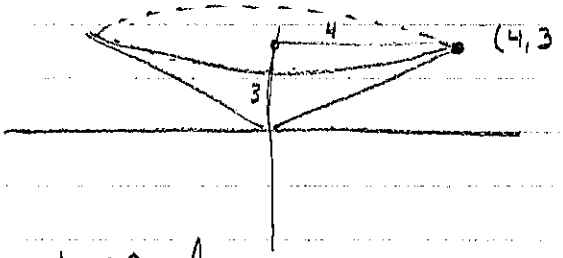


(28) a) $LA = \frac{1}{2}(2\pi r)\ell$ $\quad V = \frac{1}{3}\cdot\pi r^2 h$

$LA = \frac{1}{2}(2\pi\cdot 3)\cdot 5$ $\quad V = \frac{1}{3}\pi\cdot 3^2\cdot 4$

$LA = 15\pi$ $\quad V = 12\pi$

b)



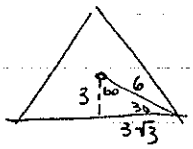
$LA = \frac{1}{2}\cdot 2\pi r\ell$ $\quad V = \frac{1}{3}\pi r^2 h$

$LA = \frac{1}{2}\cdot 2\pi(4)(5)$ $\quad V = \frac{1}{3}\pi(4^2)(3)$

$LA = 20\pi$ $\quad V = 16\pi$

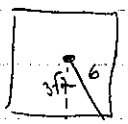
c) no

(29) a)



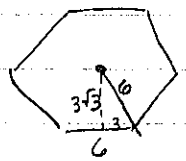
$b = 6\sqrt{3}$

b)



$b = 6\frac{\sqrt{2}}{2} = 3\sqrt{2}$

c)



$V = \left(\frac{1}{2}ap\right)(h)$ $\quad V = \left(6\sqrt{2}\right)^2\cdot 10$ $\quad V = \left(\frac{1}{2}ap\right)\cdot 10$

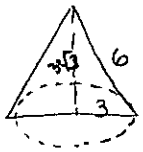
$V = \frac{1}{2}(3)(18\sqrt{3})(10)$ $\quad V = 720$ $\quad V = \frac{1}{2}(3\sqrt{3}\cdot 36)\cdot 10$

$V = 270\sqrt{3}$ $\quad V = 540\sqrt{3}$

$LA = ph$ $\quad LA = ph$ $\quad LA = ph$

$LA = (18\sqrt{3})(10)$ $\quad LA = (24\sqrt{2})(10)$ $\quad LA = 36\cdot 10$

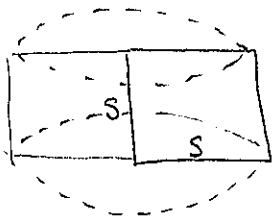
$LA = 180\sqrt{3}$ $\quad LA = 240\sqrt{2}$ $\quad LA = 360$

(30) 

$r = 3 \quad h = 3\sqrt{3} \quad V = \frac{1}{3}\pi r^2 \cdot h$

$V = \frac{1}{3}\pi(3^2)(3\sqrt{3})$

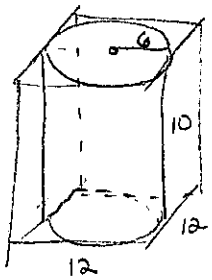
$V = 9\pi\sqrt{3}$

(31) 

$h = s \quad r = s$

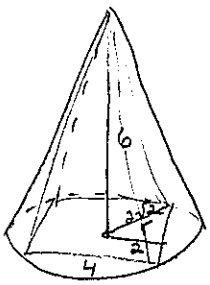
$V = \pi r^2 h$

$V = \pi \cdot s^2 \cdot s = \pi s^3$

(32) 

$V = 12^2 \cdot 10$

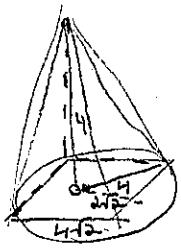
$V = 1440$

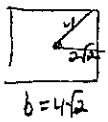
(33) 

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}(\pi \cdot (2\sqrt{2})^2) \cdot 6$

$V = 16\pi \text{ cm}^3$

(34) 

a) 

$V = \frac{1}{3}B \cdot h$

$V = \frac{1}{3}(4\sqrt{2})^2 \cdot 4$

$V = \frac{128}{3}$

b) $l_{cone}: \quad 4^2 + 4^2 = l^2$

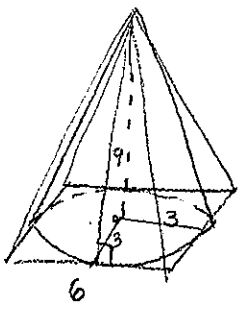
$\sqrt{32} = l = 4\sqrt{2}$

$l_{pyr}: (2\sqrt{2})^2 + 4^2 = l^2$

$\sqrt{24} = l$

$2\sqrt{6} = l$

(35)



$9^2 = 3^2 + h^2$ $\quad r = 3$

$\sqrt{72} = h$

$6\sqrt{2} = h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (3^2)(6\sqrt{2})$

$V = 18\pi\sqrt{2}$ cm^3

(36) $LA = \frac{1}{2} \cdot 2\pi r \cdot l$

$TA = \frac{1}{2} \cdot 2\pi r \cdot l + \pi r^2$

$\frac{1}{2} \cdot 2\pi r \cdot l = \frac{3}{5}\left(\frac{1}{2} \cdot 2\pi r \cdot l + \pi r^2\right)$

$\pi r l = \frac{3}{5}(\pi r l + \pi r^2$

$5\pi r l = 3\pi r l + 3\pi r^2$

$5l = 3l + 3r$

$\frac{2l}{3l} = \frac{3r}{3l}$

$\frac{2}{3} = \frac{r}{l}$