Chalkboard Examples

Classify each statement as true or false and give the definition, postulate or theorem that supports your conclusion.

1. A given triangle can lie in more than one plane. F
   Through a line and a point not in the line there is exactly 1 plane.
2. Any two points are collinear. T
   Through any 2 pts. there is exactly 1 line.
3. Two planes can intersect in only one point. F
   If 2 planes int., then their int. is a line.
4. Two lines can intersect in two points. F
   If 2 lines int., then they int. in exactly 1 pt.

Additional Answers Classroom Exercises

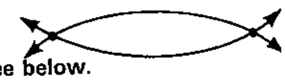
13. The ends of three legs determine a plane (the floor); the end of the fourth leg might not be in that plane.

Cultural Note

Surveying was a highly developed science in many ancient societies. Egyptian surveyors, for example, were able to measure straight lines over great distances of terrain of varying elevations.

Classroom Exercises

1. Theorem 1-1 states that two lines intersect in exactly one point. The diagram suggests what would happen if you tried to show two "lines" drawn through two points. State the postulate that makes this situation impossible. **See below.**



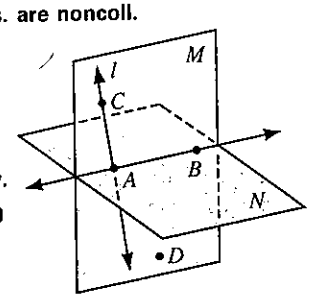
2. State Postulate 6 using the phrase *one and only one*. **See below.**
3. Reword the following statement as two statements, one describing existence and the other describing uniqueness: **Every segment has at least one midpoint. A segment has no more than one midpoint.**

   A segment has exactly one midpoint.

Postulate 6 is sometimes stated as "Two points *determine* a line."

4. Restate Theorem 1-2 using the word *determine*. **A line and a point not in the line determine a plane.**
5. Do two intersecting lines determine a plane? **Yes**
6. Do three points determine a line? **Yes; at least 1 line**
7. Do three points determine a plane? **No; unless the pts. are noncoll.**

State a postulate, or part of a postulate, that justifies your answer to each exercise.



8. Name two points that determine line l. ***C, A*; Post. 6**
9. Name three points that determine plane M. **See below.**
10. Name the intersection of planes M and N. **$\overleftrightarrow{AB}$; Post. 9**
11. Does $\overleftrightarrow{AD}$ lie in plane M? **Yes; Post. 8**
12. Does plane N contain any points not on $\overleftrightarrow{AB}$? **Yes; A plane contains at least 3 pts. not all in 1 line.**

Surveyors and photographers use a *tripod* for support.

13. Why does a three-legged support work better than one with four legs?
14. Explain why a four-legged table may rock even if the floor is level. **The legs may not be the same length and their ends may not be coplanar.**
15. A carpenter checks to see if a board is warped by laying a straightedge across the board in several directions. State the postulate that is related to this procedure. **Post. 8**
16. Think of the intersection of the ceiling and the front wall of your classroom as line l. Let the point in the center of the floor be point C.
    a. Is there a plane that contains line l and point C? **Yes**
    b. State the theorem that applies. **Theorem 1-2**

1. Through any 2 pts. there is exactly 1 line.
2. Through any 2 pts. there is one and only one line.
9. Answers may vary; any 3 of *A, B, C,* and *D*. Through any 3 noncollinear pts. there is exactly 1 plane.

Written Exercises

If there is a line and a point not on the line, then one and only one plane contains them.

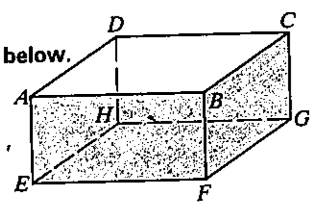
A 1. State Theorem 1-2 using the phrase *one and only one*.

2. Reword Theorem 1-3 as two statements, one describing existence and the other describing uniqueness. **See below.**

3. Planes *M* and *N* are known to intersect.
   a. What kind of figure is the intersection of *M* and *N*? **a line**
   b. State the postulate that supports your answer to part (a). **If two planes intersect, then their intersection is a line.**

4. Points *A* and *B* are known to lie in a plane.
   a. What can you say about $\overleftrightarrow{AB}$? $\overleftrightarrow{AB}$ **is in the plane.**
   b. State the postulate that supports your answer to part (a). **If 2 pts. are in a plane, then the line that contains the pts. is in that plane.**

In Exercises 5-11 you will have to visualize certain lines and planes not shown in the diagram of the box. When you name a plane, name it by using four points, no three of which are collinear.



Exs. 5–12

5. Write the postulate that assures you that $\overleftrightarrow{AC}$ exists. **See below.**
6. Name a plane that contains $\overleftrightarrow{AC}$. **ABCD**
7. Name a plane that contains $\overleftrightarrow{AC}$ but that is not shown in the diagram. **ACGE**
8. Name the intersection of plane *DCFE* and plane *ABCD*. $\overleftrightarrow{CD}$
9. Name four lines shown in the diagram that don't intersect plane *EFGH*. $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{BC}$
10. Name two lines that are not shown in the diagram and that don't intersect plane *EFGH*. $\overleftrightarrow{AC}$, $\overleftrightarrow{BD}$
11. Name three planes that don't intersect $\overleftrightarrow{EF}$ and don't contain $\overleftrightarrow{EF}$. ***ABCD, DCGH, ABGH***
12. If you measure ∠*EFG* with a protractor you get more than 90°. But you know that ∠*EFG* represents a right angle in a box. Using this as an example, complete the table.

	∠*EFG*	∠*AEF*	∠*DCB*	∠*FBC*
In the diagram	obtuse	? **rt.**	? **ac.**	? **obt.**
In the box	right	? **rt.**	? **rt.**	? **rt.**

State whether it is possible for the figure described to exist. Write *yes* or *no*.

B 13. Two points both lie in each of two lines. **No**
14. Three points all lie in each of two planes. **Yes**
15. Three noncollinear points all lie in each of two planes. **No**
16. Two points lie in a plane *X*, two other points lie in a different plane *Y*, and the four points are coplanar but not collinear. **Yes**

2. If two lines intersect, then at least one plane contains the lines. If two lines intersect, then no more than one plane contains the lines.
5. Through any 2 pts. there is exactly 1 line.

Guided Practice

Classify each statement as true or false.

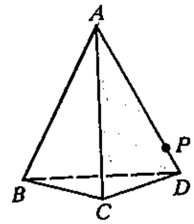
1. A postulate is a statement assumed to be true without proof. T
2. The phrase "exactly one" has the same meaning as the phrase "one and only one." T
3. Three points determine a plane. F
4. Through any two points there is exactly one plane. F
5. Through a line and a point not on the line there is one and only one plane. T

Communication Skills

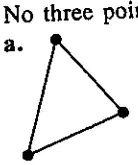
Point out to students that it takes only three noncollinea points to determine a plane but that each plane shown in the drawing contains fou labeled points. If a student chooses to name the upper base as the answer for Ex. 6, the name *plane ABCI* will be more readily understood by a person than a three-letter name such as *plane ABC.*

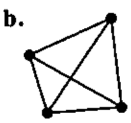
17. Points R, S, and T are noncollinear points.
   a. State the postulate that guarantees the existence of a plane X that contains R, S, and T. **Through any 3 pts. there is at least 1 plane.**
   b. Draw a diagram showing plane X containing the noncollinear points R, S, and T. **Check students' drawings.**
   c. Suppose that P is any point of $\overleftrightarrow{RS}$ other than R and S. Does point P lie in plane X? Explain. **Yes. If 2 pts. are in a plane, then the line that contains the pts. is in that plane.**
   d. State the postulate that guarantees that $\overleftrightarrow{TP}$ exists. **Through any 2 pts. there is exactly 1 line.**
   e. State the postulate that guarantees that $\overleftrightarrow{TP}$ is in Plane X. **See 17. c.**

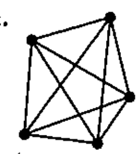
18. Points A, B, C, and D are four noncoplanar points.
   a. State the postulate that guarantees the existence of planes ABC, ABD, ACD, and BCD. **See 17. a.**
   b. Explain how the Ruler Postulate guarantees the existence of a point P between A and D.
   c. State the postulate that guarantees the existence of plane BCP. **See 17. a.**
   d. Explain why there are an infinite number of planes through $\overline{BC}$. **There are an infinite number of pts. P on $\overleftrightarrow{AD}$. For each P there exists a plane BCP.**

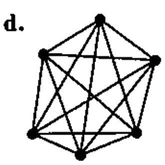


C 19. State how many segments can be drawn between the points in each figure. No three points are collinear.

a.

3 points
3 ___?___ segments

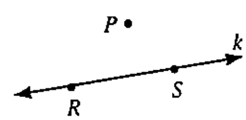
b.

4 points
6 ___?___ segments

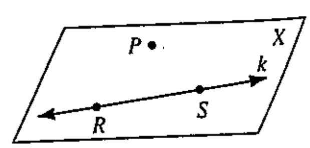
c.

5 points
10 ___?___ segments

d.

6 points
15 ___?___ segments

   e. Without making a drawing, predict how many segments can be drawn between seven points, no three of which are collinear. **21**
   f. How many segments can be drawn between n points, no three of which are collinear? $\mathbf{\frac{n(n-1)}{2}}$

20. Parts (a) through (d) justify Theorem 1-2: Through a line and a point not in the line there is exactly one plane.
   a. If P is a point not in line k, what postulate permits us to state that there are two points R and S in line k? **Post. 5**
   b. Then there is at least one plane X that contains points P, R, and S. Why? **See below.**
   c. What postulate guarantees that plane X contains line k? Now we know that there is a plane X that contains both point P and line k. **Post. 8**
   d. There can't be another plane that contains point P and line k, because then *two* planes would contain noncollinear points P, R, and S. What postulate does this contradict? **See below.**





20. b. Through any 3 pts. there is at least 1 plane.
20. d. Through any 3 noncoll. pts. there is exactly 1 plane.

Exercise Note

It is suggested that only the most capable students work on Exs. 19–20.